Math 304 **Homework 1** **Due 9/21/2005**

1. Brualdi, Chapter 2, Problems 11*, 15**, 16**, 20**, 23**.

2. ** The set M consists of nine integers none of which has prime divisors greater than 6. Prove that M has two elements whose product is a square of an integer. *Hint:* Write each of those integers as $2^k 3^m 5^n$ for some integers $k, m, n \geq 0$. Recall that those integers may be odd or even.

3. ** Mr. and Mrs. Smith invited three couples to their home. Some guests were friends of Mr. Smith, and some guests were friends of Mrs. Smith (and some may have been friends of neither or both). When the guests arrived, people who knew each other beforehand (except spouses) shook hands, and those who did not know each other just greeted each other. After all this took place, the observant Mr. Smith noticed, "How interesting. If you disregard me, there are no two people present who shook hands the same number of times".

   How many times did Mrs. Smith shook hands? (*Note:* that's Mrs. Smith, not Mr. Smith.)

4. ** (extra) Solve Problem 23 with "9" instead of "10". In other words, show that $r(3,4) \leq 9$. *Hint:* A dichotomy that works for 10 does not seem to work for 9; there seems to be the third case. Prove that this case is actually impossible by counting the number of ends of edges of each color in two different ways.

5. ** (extra) Prove that $r(3,4) > 8$. In other words, color the edges of K_8 in 2 colors, red and blue, so that there is no red K_3 or blue K_4. *Note:* It is not enough to exhibit a 2-edge-coloring that seems to have this property; you must prove that it actually does have this property. Listing all red and blue edges in each K_3 and K_4 may be too long and tedious, so see if there is an easier argument.

6. *** (extra) In Application 4, Section 2.1, what is the greatest integer n that can be substituted for 21 so that the statement of the problem remains true? *Note:* This is like a Ramsey-type problem in that its proof has two parts: 1) the statement is true for n, and 2) the statement is false for $n+1$. If you can't determine the actual integer n, try to determined (for partial credit) some nontrivial upper and lower bounds for n.

7. ***(extra) Problem 21. *Note:* It's really hard, but give it a try if you have time and inclination. Again, don't just exhibit a coloring that seems to work – prove that it does work.