Math 304 **Midterm 2** **Due 12/1/2005**

This exam is due Thursday, December 1, in class. You may consult the text for this course, your notes taken in lecture and your homework. Do not use any other books or papers or materials from a library or consult with any person other than myself. Please sign your name on your completed work and write, just above your signature, a statement to the effect that you have observed the above rules. Remember to SHOW ALL WORK.

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1. Brualdi, Chapter 8, Problems 2, 14, 16.

2. Given a integer partition $\lambda \vdash n$, $\lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k)$, prove that its conjugate is the partition λ' that has $\lambda_i - \lambda_{i+1}$ parts of each size $i \in [k]$ (when $i = k$, define $\lambda_{k+1} := 0$).

3. Brualdi, Chapter 7, Problems 8, 30(d). Extra: Problems 6, 7.

4. Prove that Fibonacci numbers satisfy the recurrence relation

$$f_n = f_k f_{n-k+1} + f_{k-1} f_{n-k} \quad (n \geq k).$$

   Remark: This may help with Problems 6 and 7.

5. Let $\{g_n\}_{n \geq 0}$ be the sequence given by

$$g_n = -n g_{n-1} + \sum_{k=1}^{n-1} \binom{n}{k} g_k g_{n-k} \quad (n \geq 2); \quad g_0 = 0, g_1 = 1.$$

   Find the exact formula for g_n.

6. Brualdi, Chapter 6, Problem 24(a). Extra: Problem 32.