This exam is due Tuesday, October 25, in class. You may consult the text for this course, your notes taken in lecture and your homework. Do not use any other books or papers or materials from a library or consult with any person other than myself. Please sign your name on your completed work and write, just above your signature, a statement to the effect that you have observed the above rules. Remember to SHOW ALL WORK.

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1. Brualdi, Problems 2.7, 3.7, 3.38, 4.16, 4.23, 4.29, 5.30.

2. Give a combinatorial proof for the formula (5.14) in the textbook: for any nonnegative integers $n$ and $k$,
$$\binom{n+1}{k+1} = \binom{0}{k} + \binom{1}{k} + \cdots + \binom{n}{k}.$$
*Hint:* Given a $(k+1)$-subset $S$ of $[n+1]$, consider the largest element in $S$.

3. Give a combinatorial proof of the following identity: for any positive integer $n$,
$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots$$
*Hint:* Show that the left-hand side is the number of even subsets of $[n]$, while the right-hand side is the number of odd subsets of $[n]$. Then give a bijection between the set of even subsets of $[n]$ and the set of odd subsets of $[n]$. (Don't forget to show that your map is, in fact, a bijection.)