

1. Brualdi, Chapter 2, Problems 11\*, 15\*\*, 16\*\*, 20\*\*, 23\*\*.
2. \* Generalize Application 9 on page 34 to show that, given nonnegative integers  $m$  and  $n$ , every sequence of  $mn + 1$  real numbers contains either a nondecreasing sequence of length  $n + 1$  or a nonincreasing sequence of length  $m + 1$ .
3. \*\* The set  $M$  consists of nine integers none of which has prime divisors greater than 6. Prove that  $M$  has two elements whose product is a square of an integer. *Hint:* Write each of those integers as  $2^k 3^m 5^n$  for some integers  $k, m, n \geq 0$ . Recall that integers can be odd or even.
4. \*\* Mr. and Mrs. Smith invited three couples to their home. Some guests were friends of Mr. Smith, and some guests were friends of Mrs. Smith (and some may have been friends of both). When the guests arrived, people who knew each other beforehand (except spouses) shook hands, and those who did not know each other just greeted each other. After all this took place, the observant Mr. Smith noticed, “How interesting. If you disregard me, there are no two people present who shook hands the same number of times”.  
How many times did Mrs. Smith shake hands? (*Note:* this is not a typo.)
5. \*\* (extra) Solve Problem 23 with “9” instead of “10”. In other words, show that  $r(3, 4) \leq 9$ . *Hint:* A dichotomy that works for 10 does not seem to work for 9; there seems to be a third case. Prove that this case is actually impossible by counting the number of edges of each color in two different ways.
6. \*\* (extra) Show that  $r(3, 4) > 8$ . In other words, color the edges of  $K_8$  in 2 colors, red and blue, so that there is no red  $K_3$  or blue  $K_4$ . *Note:* It is not enough to exhibit a 2-edge-coloring that seems to have this property; you must prove that it actually does have this property. Listing all red and blue edges in each  $K_3$  and  $K_4$  may be too long and tedious, so see if there is an easier argument.
7. \*\*\*(extra) Problem 21. *Note:* It’s really hard, but give it a try if you have time and inclination. Again, don’t just exhibit a coloring that seems to work – prove that it does work.