Part A.

1. Let a and b be nonzero integers. Prove that $d = \gcd(a, b)$ if and only if $a' = a/d$ and $b' = b/d$ are relatively prime integers.

2. (a) Use induction to prove that $0^2 + 1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ for every integer $n \geq 0$.

   (b) Use induction to prove that $0^3 + 1^3 + 2^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for every integer $n \geq 0$.

Part B.

1. Prove that $\gcd(a, bc) = 1$ if and only if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$. *Hint:* Use Theorem 0.2 several times. "Only if" part is easier. For the "if" part, express 1 as a linear combination of a and b, then c as a linear combination of a and b, then 1 as a linear combination of a and c.

2. (Chinese Remainder Theorem) Let r_1, r_2 and m_1, m_2 be nonzero integers such that $\gcd(m_1, m_2) = 1$. Prove that there is an integer n such that $n \equiv r_1 \mod m_1$ and $n \equiv r_2 \mod m_2$ simultaneously (in other words, $n = m_1q_1 + r_1$ for some integer q_1 and $n = m_2q_2 + r_2$ for some integer q_2). *Hint:* Use the fact that $\gcd(m_1, m_2) = 1$ and Theorem 0.2 to find an integer n_1 such that $n_1 \equiv 1 \mod m_1$ and $n_1 \equiv 0 \mod m_2$, and an integer n_2 such that $n_2 \equiv 0 \mod m_1$ and $n_2 \equiv 1 \mod m_2$. Then use n_1 and n_2 to find n.