1. (a) What is the smallest chromatic number for a nonplanar graph?
   (b) What is the smallest order of a planar graph G with $\chi(G) = 4$?

2. (a) Prove that the Petersen graph is not planar.
   (b) Prove that $C_3 \times C_3$ is not planar.

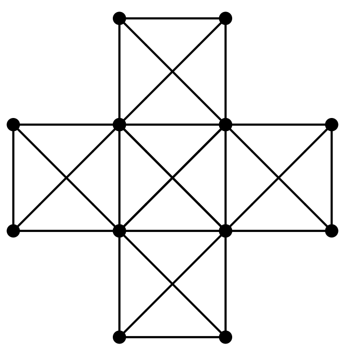
   Hint: Use Kuratowski's theorem for both parts.

3. Construct the 5-regular maximal planar graph. *Hint:* First find p, q and r.

4. Decompose K_7 into 2 planar graphs.

5. Is the graph below:
   (a) planar?
   (b) maximal planar?
   (c) If not maximal planar, either remove or add edges, or both, to make it maximal planar. (Do not remove or add vertices, and make as few changes as possible.)

G:



6. *(extra credit)* In this problem, instead of drawing graphs with as few edge crossings as possible, we will try to achieve the maximum possible number of crossings.
   (a) Prove that a simple drawing of C_6 must have ≤ 9 crossings.
   (b) Find a simple drawing of C_6 with 9 crossings. (*extra credit for a nice drawing*)

7. Molly makes knitted toys. Recently she made some objects which look like soccer balls. She knitted several pentagons and hexagons, sewed them together to form a ball, and then stuffed it. Any time two polygons were joined one entire edge from both polygons was sewn together. "It's strange" she said. "Every time I do this I use a different number of hexagons but I always end up with the same number of pentagons." "Aha!" I said. "The edges of the polygons form a normal map, and therefore the dual is ma. . . "

   How many pentagons did Molly use each time? Prove your answer.

   Hint: What are the degrees of vertices of the dual graph?