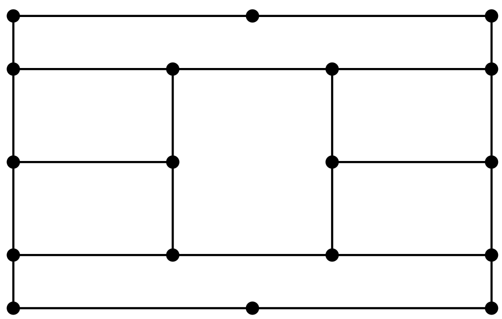
1. (a) Is the dodecahedron graph D on Figure 3.17 (Section 3.2, page 71) Hamiltonian?
   (b) Is the graph on Figure 3.21 (Section 3.2, page 75) Hamiltonian?

2. (a) Does the tennis graph have a Hamilton cycle? If yes, is there a Hamilton cycle through any given edge of the tennis graph?



The tennis graph

   (b) If the groundskeeper for such a tennis court is using his machine to put the chalk lines on the tennis court, what is the minimum number of times he must interrupt his machine?

3. Section 3.1, Problems 12 and 13, page 66.

4. Prove that if every edge of a pseudograph G lies on an odd number of cycles, then G has an Eulerian circuit. *Hint:* Consider cycles through any given vertex v of G. Sum up, over all cycles through v, the numbers of edges incident with v which lie on cycles through v. Do this in two different ways. (Each such edge is counted once for each cycle it lies on.) From the resulting equation, make a conclusion about the degree of v.

5. The cube Q_3 we have seen before is a special case of a family of graphs known as n-cubes. The n-cube Q_n has 2^n vertices each of which is labeled by a binary n-tuple (a string of length n each entry of which is either 0 or 1). Two vertices are adjacent if and only if their corresponding n-tuples differ in exactly 1 position.

   (a) What are Q_0, Q_1 and Q_2? Verify that Q_3 as defined in this problem is indeed the graph Q_3 from Problem 7, Homework 1.
   (b) For which n is Q_n Eulerian? Prove your answer.
   (c) Prove that Q_n is Hamiltonian ($n \geq 2$). *Hint:* Use induction. First, show that Q_2 is Hamiltonian, then show that for $n \geq 3$, Q_n is Hamiltonian if Q_{n-1} is Hamiltonian.

6. (extra credit) The *product* graph $G \times H$ is defined as follows. Its vertex set $V(G \times H)$ is the Cartesian product $V(G) \times V(H)$, i.e. a set of ordered pairs (g, h) such that g is a vertex of G and h is a vertex of H. Two vertices (g_1, h_1) and (g_2, h_2) of $G \times H$ are adjacent if and only if either:

- $g_1 = g_2$ and $h_1 h_2$ is an edge of H, or
- $h_1 = h_2$ and $g_1 g_2$ is an edge of G.

(a) Prove that Q_n is isomorphic to $K_2 \times Q_{n-1}$.

(b) Prove that if G and H are hamiltonian then so is $G \times H$. *Hint:* Say G has m vertices and H has n vertices. If C_m and C_n are Hamilton cycles of G and H, prove that $C_m \times C_n$ has a Hamilton cycle. (Think of $C_m \times C_n$ as a rectangular grid on a bagel.) There are two slightly different cases depending on whether m and n are odd or even. Try some small cases first to see what happens, e.g. $C_3 \times C_3$, $C_3 \times C_4$, etc.