Math 314 Homework 1 Due 2/6/2003

1. Determine which of the following sequences are graphic and, for each graphic sequence, construct a graph with that sequence.

   (a) $5, 5, 4, 4, 3, 2, 2, 1, 1$

   (b) $5, 5, 4, 4, 3, 3, 2, 2, 1, 1$

   (c) $4, 4, 4, 4, 3, 3$

   (d) $7, 6, 5, 4, 4, 3, 2, 1$

2. Prove that no graph with more than one vertex has all degrees different; that is, prove that in a degree sequence of any graph there is at least one repeated number. *Hint:* One possible proof is *by contradiction* (see appendix). Suppose there is a graph on p vertices whose degree sequence contains all different degrees. What is the greatest possible value of the highest degree? What is the smallest possible value of the lowest degree? What can you conclude if, in addition, all $p > 1$ degrees are distinct integers? Now consider the values of the highest and lowest degrees and derive a contradiction.

3. How many different (non-isomorphic) trees are there with 7 vertices? Exhibit all the different trees, and prove that any tree on 7 vertices must be isomorphic to one of your trees.

4. (a) Prove that if G is a tree, and all the degrees of vertices in G are odd, then the number of edges of G is odd.

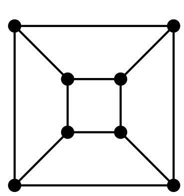
   (b) Prove by an example that the statement of part (a) is false if G is not a tree.

5. Prove that if every vertex of a graph with p vertices has degree at least $(p-1)/2$ (that's $(p-1)/2$ or more, so $(p-1)/2$ need not be an integer), then the graph is connected. Is this bound sharp? (In other words, if we change $(p-1)/2$ to $(p-2)/2$, would the statement of the problem still hold for any graph?)

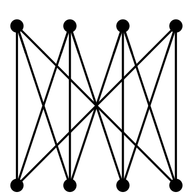
6. (a) Find a graph G with 6 vertices and 9 edges such that G has no subgraph isomorphic to K_4.

   (b) Find a graph G with 7 vertices and 9 edges such that G has no subgraph isomorphic to C_4.

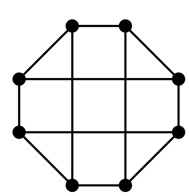
7. Which of the graphs below is *not* isomorphic to the the cube graph Q_3? Prove your answer.



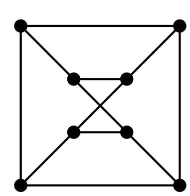
Q_3



G_1



G_2



G_3