Math 304 **Homework 2** **Due 9/30/2003**

1. Bóna, Chapter 2, Problems 16, 17, 22, 23. Extra credit: Problem 18.
2. Roberts, Section 2.7, Problems 12, 13.
3. Bóna, Chapter 3, Problems 4, 20.
4. Roberts, Section 2.8, Problem 8.

Hints:

1\. 2.16 Use the result of 1.15 which we proved in class. Note that you can plug in any nonnegative expression for each a_i.

2.17 Use the fact that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ proved in class.

2.22 Use the fact that $a_n = 10^n b_n + r_n$, where r_n is the integer represented by the last n digits of a_n, and b_n is some nonnegative integer. Find the last digit of r_n. Now consider $a_{n+1} = a_n^2$.

2.23 Given a partition of triangle T into $3n + 1$ similar triangles, cut 1 of these triangles into 4 similar ones.

2\. 2.7.12 Choose the last digit, then the first digit, then the second and third digits.

2.7.13 Use the Product Rule and the Sum Rule. In part (b), split your choices into disjoint cases, then count the number of choices in each case. In part (d), note that #("good" choices) = #(all choices) − #("bad" choices).

3\. 3.04 A counting problem plus the pigeonhole principle.

3.20 One of the ways to count suspect license plates is to choose the repeated digit, then choose the positions where it occurs, then choose the other, nonrepeated, digits in the order they occur.

4\. 2.8.8 Simplify $\frac{\binom{n}{k}}{\binom{n}{k+1}}$ as much as possible (cancel as many factors as possible). To prove unimodality, first prove that $\frac{\binom{n}{k}}{\binom{n}{k+1}} > 1$ for $k < \lfloor \frac{n}{2} \rfloor$ and $\frac{\binom{n}{k}}{\binom{n}{k+1}} < 1$ for $k > \lceil \frac{n}{2} \rceil$. Note that $\lceil \frac{n}{2} \rceil - \lfloor \frac{n}{2} \rfloor = 0$ or 1, and $\lfloor \frac{n}{2} \rfloor + \lceil \frac{n}{2} \rceil = n$.