Math 304 **Homework 1** **Due 9/16/2003**

1. Roberts, Section 8.1, Problems 1(a,b), 2(a,b), 13, 17, 24, 26 (pp. 324–325).
2. Bóna, Chapter 1, Supplementary Exercises, Problems 15, 22.
3. Five points are given in a regular triangle of side length 1. Prove that there are two among them which are at most a distance 1/2 apart from each other.
4. Prove that the sequence $32, 3232, 323232, \cdots$, has an element that is divisible by 23.
5. An ordered sequence of 13 distinct integers is such that any increasing subsequence has at most 4 terms. What is the minimum size of the largest decreasing subsequence?

Hints:

1. Problems 8.1.1 and 8.1.2 are straightforward.

8.1.13. See Example 8.2 and Problem 8.1.12 done in class.

8.1.17. 10 consecutive days is 5 consecutive sets of 2 consecutive days.

8.1.24. The pigeons are pretty obvious here, but what are the pigeonholes? Note that every integer m may be written as $m = p \cdot 2^q$, where p is an *odd* integer, and q is a nonnegative integer (i.e. every integer is a product of an odd integer and a positive integer power of 2).

8.1.26. Consider partial sums $s_i = a_1 + a_2 + \cdots + a_i$, where $1 \leq i \leq p$, and their pairwise differences.

2. 1.15 Suppose the conclusion is false, then find the lower bound on each term, then the lower bound on the sum. Derive a contradiction.

   1.22 Use the proof of the problem 8.1.24 to decide if the answer is true or false, then prove it.

3. $5 = 4 + 1$. Similar to Bóna, Example 1.3.
4. Same as Bóna, Example 1.1.
5. Use the Erdös-Szekeres theorem (as stated in class, not in the textbook).