

1. A traveling agent must visit 3 cities, each of them 3 times. In how many different ways can he do this if he is not allowed to start and finish in the same city? *Hint:* Denote the cities by A, B, C and write down the sequence in which they were visited. How many of such sequences are there? How many are forbidden? How many are allowed?
2. A track and field championship had participants from 49 countries. The flag of each participating country consisted of 3 horizontal stripes of different colors. However, no flag contained colors other than red, white, blue and green. Is it true that there were 3 participating countries with identical flags?
3. A college professor has been working for the same department for 40 years. He taught 2 courses in each semester (there are 2 semesters per year). The department offers 13 different courses. Is it sure that there were at least two semesters when this professor had identical teaching programs?
4. A cashier wants to work 5 days a week, but he wants to have at least one of Saturday or Sunday off. In how many ways can he choose the days he will work?
5. A shop can operate only if at least one sorter and at least one packager are present. There are 3 sorters and 2 packagers, and each worker is equally likely to show up for work on a given day or to stay home. Assuming that each worker decides independently whether or not to come to work, what is the probability that the shop will operate?
6. On the island of Combinatoria, all cars have license plates consisting of six numerical digits only. A witness to a crime could only give a partial description of the getaway car. In particular, she noticed that the license plate was from Combinatoria (there are no cars from elsewhere), there was only one digit that occurred more than once, and that digit occurred three times. A police officer estimated that this information will exclude more than 90 percent of all cars as suspects. Was his estimate correct?
7. (*Extra Credit*)
 - (a) (**A Goofy Judge**) A prisoner is to be tried by a 3-judge panel. Each judge decides independently whether to convict or acquit. Two “serious” judges vote for conviction with the same probability p , while the “goofy” third judge determines his choice by the toss of a (fair) coin. The prisoner is convicted if two or more judges vote to convict him. Will the prisoner be better off or worse off he were tried by a single “serious” judge instead?
 - (b) (**Serious Judges**) Suppose instead that all 3 judges are “serious” and all convict independently with the same probability p . Will the prisoner be better off being tried by a 3-judge panel (with at least 2 votes needed to convict) or by a single judge? How does that depend on the value of p ?