Math 304 **Homework 1** **Due 9/12/2002**

1. Section 8.1, Problems 1 (a,b), 2(a,b), 12, 17, 24, 25, 26 (pp. 324–325).

2. A busy airport sees 1500 takeoffs per day. Prove that there are two planes that must take off within a minute from each other.

3. Ten points are given in a square of unit size (1×1). Prove that there are two among them which are at a distance at most $\sqrt{2}/3$ from each other.

4. Prove that the sequence $1970, 19701970, 197019701970, \cdots$, has an element that is divisible by 1971.

5. An ordered sequence of 13 distinct integers is such that any decreasing subsequence has at most 3 terms. What is the minimum size of the *largest* increasing subsequence of our sequence?

Hints:

1. Problems 8.1.1 and 8.1.2 are straightforward.

   8.1.12. See Example 8.2.

   8.1.17. Similar to the 10-100 example done in class.

   8.1.24. The pigeons are relatively obvious here, but what should your pigeonholes be? Note that every integer m may be written as $m = p \cdot 2^q$, where p is an *odd* integer, and q is a nonnegative integer.

   8.1.25 Suppose there are a total of n people, and all of them have different numbers of acquaintances. What can you conclude about the people with the least and the most number of acquaintances? Derive a contradiction from your conclusions.

   8.1.26. Consider partial sums $s_i = a_1 + a_2 + \cdots + a_i$, where $1 \leq i \leq p$, and their pairwise differences.

2. Find the number of minutes in a day.

3. $10 = 3 \times 3 + 1$. Similar to the example with the regular triangle done in class.

4. Same as the problem with 2003 and $7, 77, 777, \cdots$, done in class.

5. Use the generalized version of the Erdös-Szekeres theorem.