Math 314 **Midterm** **Due 03/27/2007**

This exam is due Tuesday, March 27, in class. You may consult the text and homework solutions for this course, your notes taken in lecture and your homework. Do not use any other books or papers or materials from a library or consult with any person other than myself. Please sign your name on your completed work and write, just above your signature, a statement to the effect that you have observed the above rules. SHOW ALL WORK in problems 4–8.

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1. Complete the table by finding all nonnegative integers n (or pairs (m, n) of nonnegative integers) for which the graphs below have the following properties:

	3-regular	acyclic	bipartite	Hamiltonian	$\chi = n$	$\chi' = n$
C_n						
$K_{m,n}$						
K_n						
Q_n						
W_n						
D_n						

Note: Recall that W_n is the wheel (the cycle C_n with every vertex connected to an extra hub vertex), and D_n is the double wheel (C_n with every vertex connected to two nonadjacent hubs).

2. Complete the following: "A graph such that there is at most one path between any two vertices is a ____________."

3. Complete the following: "In a connected graph with a proper 2-coloring of the vertices, two vertices x and y have the same color if and only if ________________________."

4. Give an example of each of the following graphs:
   (a) An example of a 3-regular (also called *cubic*) graph which is not Hamiltonian.
   (b) A cubic graph G (see previous part) with $\chi(G) > \chi'(G)$.
   (c) A disconnected graph G with $\chi(G) > \Delta(G)$.

5. Consider a tree on p vertices. An internal vertex is one which has degree ≥ 2. Suppose that the tree has i internal vertices. Prove that the sum of the degrees of internal vertices is $p + i - 2$. *Hint:* What are the remaining vertices? How many of them are there?

6. How many edges does K_6 have? Is K_6 decomposable into P_3? Into P_4? Into P_6? Prove your answers.

7. Color the vertices of C_n in such a way that any two vertices at distance ≤ 2 from each other have different colors. What is the least number of colors required? *Hint:* The answer depends on whether n is divisible by 3. There is also one small special case. Of course, you can try the greedy coloring, but is it always the most economical in the end?

8. Prove that the double wheel D_n is decomposable into P_4. *Hint:* Find an initial copy of P_4 and use a turning trick.