Math 314 **Homework 6** **Due 4/24/2007**

1. Prove that $r(3,3,3) \leq 17$. *Hint:* Show that any edge-coloring of K_{17} with three colors contains a monochromatic triangle. Use the fact that $r(3,3) \leq 6$ and a similar proof.

2. Prove that any bipartite planar graph has a vertex of degree at most 3.

3. (a) Construct the 4-regular maximal planar graph. Prove that it is unique such graph.

   (b) Construct the 3-regular maximal bipartite planar graph. Prove that it is unique such graph.

   Hint: First find p, q and r in both cases.

4. The *crossing number* $\mathrm{cr}(G)$ of a graph G is the least number of crossings that a simple drawing G may have.

   (a) Prove that $\mathrm{cr}(K_5) = 1$ and $\mathrm{cr}(K_{3,3}) = 1$.

   (b) Prove that $\mathrm{cr}(C_3 \times C_3) \leq 3$.

5. The *thickness* $\theta(G)$ of a graph G is the minimum number of planar graphs into which G can be decomposed. Prove that $\theta(K_7) = 2$.

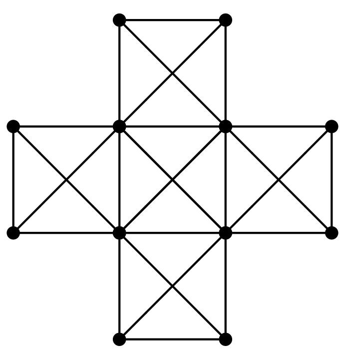
6. Is the graph below:

   (a) planar?

   (b) maximal planar?

   (c) If not maximal planar, either remove or add edges, or both, to make it maximal planar. (Do not remove or add vertices, and make as few changes as possible.)

G:



7. In this problem, instead of drawing graphs with as few edge crossings as possible, we will try to achieve the maximum possible number of crossings.

   (a) Prove that a simple drawing of C_6 must have ≤ 9 crossings.

   (b) Find a simple drawing of C_6 with 9 crossings. (*extra credit for a nice drawing*)

8. *(extra credit)* Draw K_7 on the surface of a torus (the shape of a Lifesavers candy) so that it has no edge crossings.

   Hint: You don't have to submit the actual torus. Just draw K_7 on the square where the top and bottom edges as well as the left and right edges are identified (considered to be glued together). So top and bottom edges is the same line, left and right edges is the same line, and all four corners is the same point. On your drawing, label the vertices and all important pairs of points along the edges that are identified. A nice, symmetric drawing is the best, as always.