Math 314 **Homework 5** **Due 4/12/2007**

1. For which n is Q_n Eulerian? Prove your answer.
2. Recall the tennis graph from a previous homework. If the groundskeeper for such a tennis court is using his machine to put the chalk lines on the tennis court, what is the minimum number of times he must interrupt his machine? Prove your answer.
3. Construct the labeled tree whose Prüfer code is $(4, 4, 1, 3, 3, 1)$. To show your solution, also label the edges of the tree in the order you draw them when constructing the tree.
4. (a) Prove that $\kappa(Q_n) = n$. *Hint:* Try a proof by induction. One possible strategy is to prove that deletion of any set of $n-1$ vertices leaves the graph connected.
   (b) Prove that $\kappa(C_3 \times C_3) = 4$. *Hint:* For the more nontrivial part of the proof, show that deletion of any 3 vertices leaves the graph connected. There are several distinct cases to consider.
5. Prove that the Ramsey number $r(3, 4) = r(K_3, K_4) > 8$ by decomposing the complete graph K_8 into two graphs G (red) and $\overline{G}$ (blue) such that G has no K_3 as a subgraph and $\overline{G}$ has no K_4 as a subgraph. Prove that both subgraphs indeed have the desired properties. *Hint:* Try to find a nice decomposition where both subgraphs are regular and rotationally symmetric.
6. *(extra credit)* Let G be a connected graph on p vertices and let $m < p$. Prove that if for all distinct nonadjacent vertices u and v we have $\deg(u) + \deg(v) \geq m$, then G contains a path of length m. *Hint:* Consider the longest path in G and its end vertices. Use an argument similar to that in the proof of the Ore's theorem.