Math 314 **Homework 2** **Due 2/15/2007**

1. How many different (non-isomorphic) trees are there with 7 vertices? Exhibit all the different trees, and prove that any tree on 7 vertices must be isomorphic to one of your trees.

2. (a) Prove that if G is a tree, and all the degrees of vertices in G are odd, then the number of edges of G is odd.

   (b) Prove by example that the statement of part (a) is false if G is not a tree.

3. Prove that if every vertex in a graph G on p vertices has degree at least $(p-1)/2$ (i.e. $(p-1)/2$ or more, so $(p-1)/2$ need not be an integer), then the graph is connected. Is this bound *sharp* (in other words, if we change $(p-1)/2$ to $(p-2)/2$, would the statement of the problem still hold for any graph)? *Hint:* Prove that any two vertices in G are neighbors or have a common neighbor.

4. (a) Find a graph G with 6 vertices and 9 edges such that G has no subgraph isomorphic to K_4.

   (b) Find a graph G with 7 vertices and 9 edges such that G has no subgraph isomorphic to C_4.