

1. (a) not graphic (sum of the degrees is odd). (b) graphic. (c) graphic. (d) graphic.
2. The remaining degree must be between 0 and  $7 - 1 = 6$  and odd since we must have an even number of odd degrees. This yields three possibilities: 1, 3 and 5. Applying Havel-Hakimi algorithm, we see that only degree 3 yields a graphic sequence.
3. Let  $G$  be any graph on  $p$  vertices, and let  $\Delta$  and  $\delta$  be the maximum and minimum degree of  $G$ , respectively. Then  $\Delta \leq p - 1$  and  $\delta \geq 0$ . Suppose the degree sequence of  $G$  has all degrees different. Then it contains  $p$  integers (as many as there are vertices) between 0 and  $p - 1$ . But there are exactly  $p$  integers between 0 and  $p - 1$ , therefore  $\Delta(G) = p - 1$  and  $\delta(G) = 0$ . Hence, our graph must contain a vertex of degree 0, so a vertex of degree  $\Delta(G)$  can be adjacent to at most  $p - 2$  vertices. In other words,  $\Delta \leq p - 2$ . This is a contradiction. Thus, the degree sequence of any graph must contain a repeated element.
4. Graph  $G_3$  is the only one not isomorphic to the cube graph  $Q_3$ , since it contains an odd cycle  $C_5$  as a subgraph, which  $Q_3$  does not (see below). To see that the other two graphs are isomorphic to  $Q_3$ , number the vertices so as to have the same set of edges. To see that  $Q_3$  does not contain  $C_5$  note that we can color the vertices of  $Q_3$  with two colors so that no two vertices of the same color are adjacent. (This is especially easy to do with  $Q_3$  drawn as  $G_1$  if we color the top vertices with one color and the bottom vertices with the other color.) Obviously, we cannot do the same with  $C_5$ , so  $C_5$  cannot be a subgraph of  $Q_3$ .