Math 314 **Homework 1** **Due 1/30/2007**

1. Determine which of the following sequences are graphic and, for each graphic sequence, construct a graph with that sequence.

   (a) $5, 5, 4, 4, 3, 2, 2, 1, 1$

   (b) $5, 5, 4, 4, 3, 3, 2, 2, 1, 1$

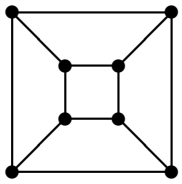
   (c) $4, 4, 4, 4, 3, 3$

   (d) $7, 6, 5, 4, 4, 3, 2, 1$

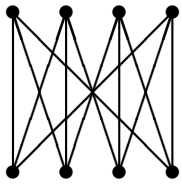
2. A graph on 7 vertices has a vertex of degree 6, a vertex of degree 5, a vertex of degree 4, a vertex of degree 3, a vertex of degree 2, and a vertex of degree 1. Determine the degree of the remaining vertex.

3. Prove that no graph with more than one vertex has all degrees different; that is, prove that in a degree sequence of any graph there is at least one repeated number. *Hint:* One possible proof, *by contradiction*, is as follows. Suppose there is a graph on p vertices whose degree sequence contains all different degrees. What is the greatest possible value of the highest degree? What is the least possible value of the lowest degree? What can you conclude if, in addition, all $p > 1$ degrees are distinct integers? Now consider the values of the highest and lowest degrees and derive a contradiction.

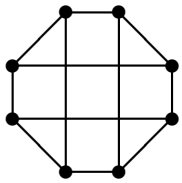
4. Which of the graphs G_1, G_2, G_3 below is *not* isomorphic to the the cube graph Q_3 (on the left)? Prove your answer.



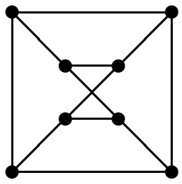
Q_3



G_1



G_2



G_3